T TEST

Part 2

b. Analysis of independent sample experiment:

The way to estimate the difference in population means μ_1-μ_2 is to

- Calculate the means of the two samples representing the two populations, x_1 and x_2 and use them to
- Estimate the difference between the two population means.
- **The standard error of the difference between the two means is needed for the t test.**

- **A pooled estimate of the variance in the two samples is calculated, with the assumption that the variance in the two populations is the same (i.e. $\sigma_1 = \sigma_2$).**
- **The computation procedure in the case when the size of the two samples is the same, i.e. when $n_1 = n_2$, differs from that when size of sample differs, i.e. when $n_1 \neq n_2$.**

A. Comparing two samples of equal variance equal size ($\sigma 1=\sigma 2$) and ($n1= n2$)

- The comb weights of two lots of 15 day-old male chicks, one receiving sex hormone A (testosterone), the other C (dehydroandrosterone).
- Day old chicks, 11 in number were assigned at random to each of the two treatments. To distinguish between the two lots, which were caged together, the heads of the chicks were stained red and purple, respectively. The individual comb weights are presented below:

	Hormone A	Hormone C
	57	89
	120	30
	101	82
	137	50
	119	39
	117	22
	104	57
	73	32
	53	96
	68	31
	118	88
$\sum x$	1067	616
N	11	11
$\bar{x}$	97	56
$\sum x^2$	111971	42244

A. Comparing two independent samples of equal size :

After calculating the means and variances of the two samples, the pooled variance is estimated by:

$$S_P^2 = \frac{(S_1^2 + S_2^2)}{2}$$

Where:

S_P^2 **= the pooled variance**

S_1^2 **= variance of sample 1**

S_2^2 **= variance of sample 2**

The standard error of the difference between the two sample means is obtained by:

$$S_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{2\sigma_P^2}{n}}$$

The appropriate statistic

for testing H_0: $\mu_1 = \mu_2$ is:

$$t' = \frac{\bar{x}_1 - \bar{x}_2}{S_{\bar{x}_1 - \bar{x}_2}}$$

This is to be compared to the t-tabulated (two tail) at d. f. = 2(n-1).

Steps

$$S_p^2 = \frac{S_1^2 + S_2^2}{2}$$

$$S^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1}$$

$$S_1^2 = \frac{111971 - \frac{(1067)^2}{11}}{10} = 847.2$$

$$S_2^2 = \frac{42244 - \frac{(616)^2}{11}}{10} = 774.8$$

$$S_p^2 = \frac{S_1^2 + S_2^2}{2}$$

$$S_p^2 = \frac{(847.2 + 774.8)}{11} = 811$$

$$S_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{2(S_p^2)}{n}}$$

$$S_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{2(811)}{11}} = 12.14$$

$$t = \frac{97 - 56}{12.14} = 3.38$$

- t- tabulated (two tail)

- At d.f. . 2 (n-1)=20

- At level of significance

0.05 = 2.086

0.01 = 2.845

- t-calculated > t-tabulated

∴ There is a significance difference.

□ Thus, the null hypothesis is rejected and the alternative hypothesis is accepted.

- And there is enough evidence that hormone A causes significantly heavier combs than hormone C.

b. Comparing two samples of equal variance and unequal Size ($\sigma_1=\sigma_2$) and ($n_1 \neq n_2$):

- In planned experiments equal numbers are preferable, being simpler to analyze and more efficient. But, equality is sometimes impossible or inconvenient to attain.
- Two lots of chicks from two batches of eggs treated differently nearly always differ in the number of birds hatched. Occasionally when a new treatment is in short supply an experiment with unequal is set up deliberately.

- Unequal numbers occur also in experiments because of accidents and losses during the experiment in such cases the investigator should always consider whether any loss represents a failure the treatment rather than an accident that is to be blamed on the treatment. Such situations of course require careful judgment.

- **The statistical analysis for samples of unequal sizes follows almost exactly the same pattern as that for groups of equal size. We also assume that the variance is the same in both populations.**

The statistic t is estimated by

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S_{\bar{X}_1 - \bar{X}_2}}$$

The pooled variance S_p^2 is estimated by

$$S_p^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

However, **this weighted pooled variance** is obtained by calculating **a weighted mean of the variances of the two samples** as follows:

$$S_P^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

The standard error of the difference between the two means is obtained by:

$$S_{\bar{X}1-\bar{X}2} = \sqrt{\frac{S_P^2}{n_1} + \frac{S_P^2}{n_2}} = \sqrt{S_P^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

The suitable statistic t is calculated as:

$$t' = \frac{\bar{x}_1 - \bar{x}_2}{S_{\bar{x}1-\bar{x}2}}$$

This value is compared to the t-tabulated (two-tail).

The number of degrees of freedom is:

$$d.f. = n_1 + n_2 - 2$$

- The standard error of the difference between the two means:

$$S_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$$

- **The tabulated t at degrees of freedom is ($n_1 + n_2 - 2$)**

Example:

- The fol1owing table represents the gains in weights of two lots of rats of different sizes under two different diets (gain in g).

	High protein	Low protein
	134	70
	104	118
	124	85
	107	107
	113	132
	97	94
	146	101
	119	
	161	
	83	
	129	
	123	
$\sum x$	1440	707
N	12	7
$\bar{x}$	120	101
$\sum x^2$	177832	73959

Steps

$$S^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$$

$$S_1^2 = \frac{177832 - \frac{(1440)^2}{12}}{11} = 457.45$$

$$S_1^2 = \frac{73959 - \frac{(707)^2}{7}}{6} = 425.33$$

The pooled variance $S_p^2 = \frac{(457.45)11 + (425.33)6}{12 + 7 - 2} = 446.12$

$$S_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{S_p^2}{n_1}+\frac{S_p^2}{n_2}} \quad S_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{446.12}{12}+\frac{446.12}{7}} = \sqrt{37.18+63.73} = \sqrt{100.91} = 10.04$$

$$\text{The statistic t is estimated by} = \frac{\bar{X}_1 - \bar{X}_2}{S_{\bar{X}_1-\bar{X}_2}} \qquad \text{Calculated t} = \frac{120-101}{10.04} = 1.89$$

- Tabulated at d.f. = $(n_1+n_2-2) = (12+7-2) = 17$ ($t_{0.05} = 2.110$)
- As the **t** calculated is less than **t** tabulated the difference between the two means is non-significant. In other words the null hypothesis ($H_o: \mu_1=\mu_2$) is accepted at 0.05 level.

THANKS